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Technical Comments

Comment on "Aircraft with Single-Axis Aerodynamically Deployed Wings"

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IN Ref. 1, the authors raise the question: "Is there an axis fixed in both [wing] B and [fuselage] A, such that, if B is rotated around the indicated axis relative to A an amount θ, starting in the [stored] configuration . . . , then B ends up having the [deployed] configuration . . . ?" They say they were able to show that such an axis exists.

This question is but a special case of one that Leonard Euler raised and solved long ago.[2] A famous theorem, due to Euler, states that any two orientations can be bridged by a single rotation around a fixed axis.

The angle of this rotation may be extracted from the orthogonal matrix R describing the relative orientation by

$$\cos\theta = \tfrac{1}{2}(R_{11} + R_{22} + R_{33} - 1) \tag{1}$$

The components of a unit vector $\boldsymbol{e}$ along the rotation axis are given by

$$e_1 = \frac{R_{32} - R_{23}}{2\sin\theta} \tag{2}$$

$$e_2 = \frac{R_{13} - R_{31}}{2\sin\theta} \tag{3}$$

$$e_3 = \frac{R_{21} - R_{12}}{2\sin\theta} \tag{4}$$

Equation (1) determines two angles. In general, one is smaller than π and the other larger than π. Whichever angle is used, the required rotation is in the right screw sense around the vector $\boldsymbol{e}$. See Ref. 3 for further details.

Euler's theorem states that the axis of rotation is unique. How is it, then, that in Ref. 1, two axes are found? This is because Ref. 1 addresses a rectangular wing and does not distinguish between the root and the tip. When a more general wing is admitted and the root and tip are predefined, only one axis exists for the deployment with one angle smaller than π (120 deg in the case studied in the reference), and one larger than π (240 deg for the case in the reference). Case DL has the wing stored with the tip forward, and case UL with the tip aft.

Both cases assume that the wing is stored with the top surface on the outside. If storage with the bottom surface on the outside was considered, two new deployment axes would become available. If the stored position were rotated to lie partway down the fuselage and/or the deployed position were allowed sweep or dihedral, further variations in deployment geometry would result with the angle no longer being 120 deg. Also, taper of the fuselage or of the wing, with the wing still stored hugging the fuselage, would cause the deployment angle to deviate from 120 deg.

In conclusion, the designer is free to select the stored orientation and the deployed orientation to advantage. Once this is done, Eqs. (1–4) determine the axis and angle of deployment. The analysis of Djerassi and Kotzev can then be applied.

References

[1]Djerassi, S., and Kotzev, S., "Aircraft with Single-Axis Aerodynamically Deployed Wings," *Journal of Aircraft*, Vol. 32, No. 2, 1995, pp. 343–348.

[2]Euler, L., *Novi Commentarii Academiae Petropolitanae*, Vol. 15, 1770, pp. 13–15, 75–106.

[3]Katz, A., *Rigid Vehicle Dynamics*, Univ. of Alabama Academic Publishing Service, Tuscaloosa, AL, 1991, 1995.

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Reply by the Author to A. Katz

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THE author of Ref. 1 draws attention to a question raised in Ref. 2, and to its relation to a theorem by Euler. Moreover, he concludes that Eqs. (1–5) in Ref. 2 were obtained under the assumption that the wing of the aircraft discussed in Ref. 2 is rectangular. It will be shown here that, in fact, such an assumption has not been made.

To this end, suppose wing B undergoes a simple rotation in fuselage A (i.e., a rotation about an axis L fixed in both A and B) of the aircraft described in Fig. 1 of Ref. 2. Let $\boldsymbol{a}_i$ and $\boldsymbol{b}_i$ (i = 1, 2, 3) be two sets of dextral, mutually perpendicular unit vectors fixed in A and in B, respectively, as shown in Fig. 1. Choosing $\boldsymbol{a}_i = \boldsymbol{b}_i$ (i = 1, 2, 3) when B is deployed, note that, from an engineering point of view, the deployed configuration of B in A is unique, and that there are four admissible stored

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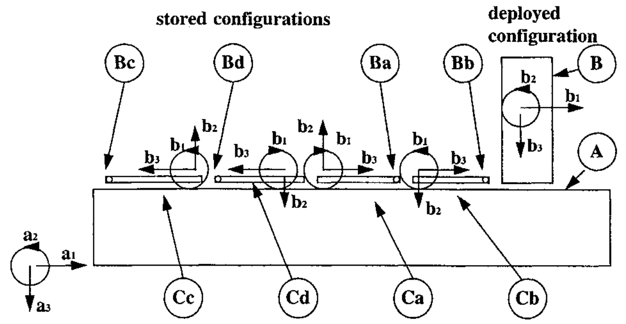

Fig. 1 Stored and deployed configurations.

Table 1 Admissible solutions

Configuration	$e_1\sqrt{3}$	$e_2\sqrt{3}$	$e_3\sqrt{3}$	θ
Ca	1	−1	1	$2/3\pi$
Cb	−1	−1	−1	$2/3\pi$
Cc	1	1	−1	$2/3\pi$
Cd	−1	1	1	$2/3\pi$

configurations of B in A, namely, the ones called *Ca*, *Cb*, *Cc*, and *Cd* in Fig. 1. In connection with each of these, one may pose the question of Ref. 2, namely, "Is there an axis . . . ?" In accordance with Euler's theorem, the answer is positive when applied to each of *Ca*, *Cb*, *Cc*, and *Cd*, and involves a point P fixed in both A and B, a line L passing through P and fixed in both A and B, a unit vector $\boldsymbol{\lambda}$ parallel to L, and an angle θ indicating the measure of rotation of B in A about L from the stored configuration to the deployed configuration. Equations (1–4) in Ref. 1 (and, similarly, the results of Ref. 3, originally used to conduct this analysis) enable the calculation of θ and e_i, defined as $e_i = \boldsymbol{\lambda}\cdot\boldsymbol{a}_i$ $(i = 1, 2, 3)$. For example, Eq. (1) leads, in connection with the *Ca* configuration (and, in fact, in connection with each of the indicated configurations), to $\theta = \pm(2/3\pi + n\pi)$ $(n = 0, 1, \ldots)$, and the value of θ appearing in Table 1 is chosen from an engineering standpoint. Moreover, in connection with the *Ca* configuration, R_{ij} $(i, j = 1, 2, 3)$, the direction cosines relating the deployed configuration of B to the *Ca* (stored) configuration are $R_{11} = 0$, $R_{12} = -1$, $R_{13} = 0$, $R_{21} = 0$, $R_{22} = 0$, $R_{23} = -1$, and $R_{31} = 1$, $R_{32} = 0$, $R_{33} = 0$, obtained with the aid of Fig. 1, by inspection; and when substituted in Eqs. (2–4) in Ref. 1, lead to the values reported in the first row of Table 1. Lastly, points *Ba*, *Bb*, *Bc*, and *Bd* shown in Fig. 1 play the role of P in connection with configurations *Ca*, *Cb*, *Cc*, and *Cd*, respectively.

Keeping in mind that the wing has to be aerodynamically deployed, and assuming that the forward velocity of the aircraft is essentially in the $-\boldsymbol{a}_1$ direction, as described in Ref. 2, one must rule out configurations *Cc* and *Cd* as inadequate for aerodynamical deployment; and is left with configurations *Ca* and *Cb*. In Ref. 2 these are called *LD* and *UD* configurations, respectively, and are the ones associated with Eqs. (1–5).

In conclusion, the choice of the *LD* and *UD* configurations in Ref. 2 as admissible is the outcome of an engineering elimination process, in which all possible results suggested by the theoretical analysis have been considered; and not because the wing was assumed to be rectangular.

References

[1]Katz, A., "Comment on 'Aircraft with Single-Axis Aerodynamically Deployed Wings,'" *Journal of Aircraft*, Vol. 33, No. 5, 1996, p. 1029.

[2]Djerassi, S., and Kotzev, S., "Aircraft with Single-Axis Aerodynamically Deployed Wings," *Journal of Aircraft*, Vol. 32, No. 2, 1995, pp. 343–348.

[3]Kane, T. R., Likins, P. W., and Levinson, D. A., *Spacecraft Dynamics*, McGraw–Hill, New York, 1983.

Comment on "Analytic *V* Speeds from Linearized Propeller Polar"

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IN a recent Technical Note, Lowry[1] presented several equations that had been previously given by Laitone.[2] For example, Lowry's Eq. (12) for the maximum level flight speed V_m can be mathematically clarified as follows when $T = D$:

$$
\begin{gathered}
T = E + FV^2 \qquad D = GV^2 + H/V^2 \\
(G - F)V^4 - EV^2 + H = 0 \\
V_m^2 = \frac{1}{2(G - F)}\left[E + \sqrt{E^2 - 4(G - F)H}\right]
\end{gathered}
\tag{1}
$$

Since Lowry[1] defines $(G - F) > 0$, Eq. (1) is identical to Lowry's Eq. (12), and is identical to Laitone's[2] Eq. (11), if one notes that F must be negative for any fixed-pitch propeller since its thrust must decrease as the velocity increases.

As shown by Laitone,[2] the thrust for any fixed-pitch propeller can be approximated by the theoretical relation

$$
T = \frac{P_p}{V_p}\left[\frac{3}{2}\phi - \frac{\sigma}{2}\left(\frac{V}{V_p}\right)^2\right] = \frac{P_{av}}{V} \tag{2}
$$

where V_p is the velocity corresponding to peak or maximum power available $(P_{av})_{max} = P_p$. Therefore, this ideal fixed-pitch propeller has

$$
F = -(\sigma/2)(P_p/V_p^3) \quad \text{and} \quad T = 0 = P_{av} \quad \text{when} \quad V_0 = \sqrt{3\phi/\sigma}V_p \tag{3}
$$

The altitude correction factor $\phi \le \sigma = \rho/\rho_0$, used by Lowry,[1] is discussed in Refs. 3 and 4, which show the effect of various approximations to ϕ, and compare the performance predictions from Eq. (2) with various methods.

Similarly, for $F = -\sigma P_p/2V_p^3$ and $\cos\alpha = 1$, Lowry's[1] Eq. (14) for the speed for fastest rate of climb is identical to Laitone's[2] Eq. (13), and his Eq. (16) for the speed for maximum angle of climb is identical to Laitone's Eq. (20). Assuming $\cos\alpha \approx 1$ is justified for any conventional airplane with a fixed-pitch propeller since the thrust vector remains nearly parallel to the velocity vector for $C_L < 1$. Lowry[1] also states, following his Eq. (18), that $F < 0$ would be "intuitively correct." However his Eqs. (5) and (18) do not give a realistic estimate of the increase in $|F|$ when the blade pitch angle β is decreased. Most analyses show that $F \sim 1/\beta^3$, and this is in agreement with Eq. (3) since a decrease of β directly decreases V_p. The dependence of V_p upon β can be established from Eq. (3) by

$$
T = 0 \quad \text{when} \quad [\beta_0 - \tan^{-1}(V_0/r\omega)] \approx \alpha = 0
$$

Therefore, when $\phi = \sigma = 1$, Eq. (2) is defined for any selected β_0 and engine power (bhp) by the following:

$$
V_0 = r\omega\tan\beta_0, \qquad V_p = V_0/\sqrt{3}, \qquad P_p = \eta\,(\text{bhp}) \tag{4}
$$

The accuracy of this approximation for V_p and P_p in Eq. (2)

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